AD-A146 680 THE APPLICATION OF QUEUEING THEORY TO COMMUNICATIONS NETWORKS -- A REVIEW(U) ROYAL SIGNALS AND RADAR ESTABLISHMENT MALVERN (ENGLAND) T E KING MAR 84 1/1

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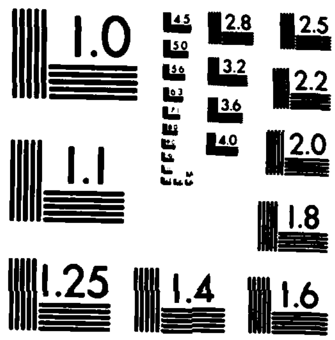

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ROYAL SIGNALS & RADAR ESTABLISHMENT

THE APPLICATION OF QUEUEING THEORY TO COMMUNICATIONS NETWORKS – A REVIEW

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PROCUREMENT EXECUTIVE,
MINISTRY OF DEFENCE,
RSRE MALVERN,
WORCS.

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THE APPLICATION OF QUEUEING THEORY TO COMMUNICATIONS NETWORKS - A REVIEW

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SUMMARY

The application of queueing theory to the performance analysis of store-and-forward communications networks is described. Some basic definitions and results in probability theory are reviewed, and the important concept of the Markov process is introduced. Applications of Markov and queueing theory to the study of networks and network components in equilibrium are taken from a wide range of recent research literature, with the emphasis placed upon the formulation of the mathematical model, rather than its solution.

The author feels that this Review should prove invaluable to mathematicians who wish to gain an appreciation of the power (and limitations) of an analytical technique that occupies an important position in current communications research.

1. INTRODUCTION

From its beginnings twenty years ago with the ARPANET experimental network, the evolution of packet-switched communications networks has been well documented [1,3]. Landmarks in this evolution are:

(i) By the end of the 1960's, plummeting computer hardware costs implied that it was cheaper to allocate communications resources dynamically (i.e. as a switched network) than to install extra resources such as point-to-point links.

(ii) During the 1970's, the introduction of standard protocols and the concept of a layered architecture made it possible (in principle) for networks to be designed in a logical way and to be interconnected.

Although the individual protocols for data link control, flow control and congestion control may be easily understood, we cannot as yet predict the consequences of their interactions in a network subject to errors and random loading over a period of time. The results of this ignorance are deadlocks and unnecessary degradation in the network performance.

In an attempt to understand network behaviour, much work has been done in the mathematical analysis field by American groups: Professor Leonard Kleinrock (University of California, Los Angeles) and Dr. Martin Reiser (IBM) are notable examples. Several reviews of analytical work exist [4]-[8].

Because the traffic offered to a network (and perhaps also its routing within the network) is random, we are forced to resort to a description of the network characteristics (such as throughput, delay and lost traffic) in terms of probability distributions. (In fact, the idea of network resource sharing is based essentially on a result in probability theory, namely the law of large numbers: although the traffic from an individual user may be bursty in nature, the total amount of traffic from many users is likely to be smooth as a function of time, thus leading to an efficient use of resources.) Markov theory is a powerful method for describing the stochastic (random) nature of communications networks, as it leads in many cases to tractable models; nevertheless it can only be applied to fairly simple situations, so other techniques such as the diffusion approximation are being employed to treat more complicated models in an approximate manner. Finally, control theory has been used to investigate the control of networks, although again there are considerable difficulties in obtaining a tractable but realistic model.

In Chapter 2, various definitions and results in probability theory are reviewed. These are needed in order to understand Chapter 3, which concerns itself with the theory of stochastic processes, including Markov processes, queueing theory and renewal theory. Chapter 4 describes the applications of Markov processes and queueing theory to the performance evaluation of protocols and network components, with examples taken from current research literature. In Chapter 5 the limitations of queueing theory are discussed with reference to blocking and priority in networks. Chapter 6 reviews the use of queueing theory and related approximations in network analysis.

(The material in Chapters 2 and 3 is taken in part from Kleinrock's excellent books on queueing theory [2,3].)

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2. REVIEW OF PROBABILITY AND TRANSFORM THEORY

2.1 The rules of probability

We consider a model which consists of three entities:

(i) The SAMPLE SPACE S, which is the set of mutually exclusive exhaustive outcomes of the model of an experiment. Each member of S is a SAMPLE POINT.

(ii) A family of EVENTS E, where each event is a set of sample points. An event corresponds to a real result.

(iii) A PROBABILITY MEASURE P which maps the events A defined on S into the set of real numbers. It satisfies $0 \leq P[A] \leq 1$, $P[S]=1$. Moreover, if A and B are mutually exclusive, then $P[A \cup B]=P[A]+P[B]$.

The CONDITIONAL PROBABILITY of A occurring given that B occurred is defined by

$$P[A|B] \equiv P[AB]/P[B] \quad (\text{ for } P[B] \neq 0).$$

A and B are STATISTICALLY INDEPENDENT iff $P[AB] = P[A]P[B]$.

Let $\{A_i\}$ be a set of mutually exclusive (i.e. $A_i A_j = \emptyset$ for all i,j, where $\emptyset$ is the null set) and exhaustive (i.e. $A_1 \cup A_2 \cup \ldots \cup A_n = S$) events. Then this set satisfies:

(i) Theorem of total probability:

$$P[B] = \sum_{i=1}^{n} P[A_i B] \quad (= \sum_{i=1}^{n} P[B|A_i]P[A_i] \text{ from definition of conditional probability})$$

(ii) Bayes' theorem:

$$P[A_i|B] = P[B|A_i]P[A_i] \Big/ \sum_{j=1}^{n} P[B|A_j]P[A_j]$$

Finally, the numbers of permutations and combinations respectively of N objects taken K at a time are given by

$${}^{N}P_{K} = N!/(N-K)! \quad ; \quad {}^{N}C_{K} = \binom{N}{K} = N!/K!/(N-K)!$$

2.2 Random variables

(a) A random variable is one whose value X(w) depends upon the outcome w of a random experiment. Define the event $[X=x] = \{w: X(w)=x\}$. Then $P[X=x]$ is the probability that $X(w)=x$, which is the sum of the probabilities associated with each outcome w for which $X(w)=x$.

The PROBABILITY DISTRIBUTION FUNCTION (PDF) of X is

$$F_X(x) \equiv P[X \leq x], \text{ where } [X \leq x] \equiv \{w: X(w) \leq x\}$$

The PROBABILITY DENSITY FUNCTION (pdf) of X is $f_X(x) = dF_X(x)/dx$. The probability of an event occurring between x and x+dx is given by $f_X(x)dx$.

Thus we have

$$P[a<X\leq b] = \int_a^b f_X(x)dx = F_X(b) - F_X(a)$$

The joint PDF and pdf for two random variables X and Y are

$$F_{X,Y}(x,y) \equiv P[X\leq x, Y\leq y] \;; \quad f_{X,Y}(x,y) \equiv d^2F_{X,Y}(x,y)/dx/dy$$

The marginal pdf for X is then defined by $f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)dy$.

(b) Random variables $X_1, X_2, \ldots, X_n$ are independent iff

$$f_{X_1,\ldots,X_n}(x_1,\ldots,x_n) = f_{X_1}(x_1)\ldots f_{X_n}(x_n)$$

The conditional pdf is defined by

$$f_{X|Y}(x|y) \equiv d/dx\; P[X\leq x|Y=y] = f_{X,Y}(x,y)/f_Y(y)$$

The pdf of the sum of two independent random variables X_1 and X_2 can be shown to obey the following rule:

$$f_{X_1+X_2}(y) = \int_{-\infty}^{\infty} f_{X_1}(y-x_2) f_{X_2}(x_2)dx_2 \equiv f_{X_1}(y) \circledast f_{X_2}(y)$$

where $\circledast$ is the CONVOLUTION OPERATOR.

In general, $f_Y(y) = f_{X_1}(y) \circledast \ldots \circledast f_{X_n}(y)$ for $Y = X_1 + X_2 + \ldots + X_n$

2.3 Expectation

(a) The expectation of a random variable X(w) is given by:

$$E[X] \equiv \overline{X} \equiv \int_{-\infty}^{\infty} xf(x)dx = \int_0^{\infty}[1-F(x)]dx - \int_{-\infty}^{0} F(x)dx$$

where the final expression is obtained by integration by parts.

If Y is a function of X, i.e. $Y = g(X)$, then

$$E_Y[Y] = \int_{-\infty}^{\infty} y f_Y(y)dy$$

If $f_X(x)$ is known, but not $f_Y(y)$, then we may use the fundamental theorem of expectation:

$$E_Y[Y] = E_X[g(X)] = \int_{-\infty}^{\infty} g(x) f_X(x)dx$$

It is always true that $\overline{X+Y} = \overline{X} + \overline{Y}$. If X and Y are independent, then $\overline{XY} = \overline{X}\,\overline{Y}$.

(b) The nth moment of X is $\overline{X^n} \equiv \int_{-\infty}^{\infty} x^n f_X(x)dx$.

The nth central moment is $\overline{(X-\overline{X})^n} \equiv \int_{-\infty}^{\infty} (x-\overline{X})^n f_X(x)dx$.

An important central moment is the VARIANCE $\sigma_X^2 \equiv \overline{(X-\overline{X})^2} = \overline{X^2} - (\overline{X})^2$. σ_X is the STANDARD DEVIATION; the coefficient of variation is $C_X \equiv \sigma_X/\overline{X}$. If $Y = X_1 + X_2 + \ldots + X_n$ and the X_i are independent, then $\sigma_Y^2 = \sum_i \sigma_{X_i}^2$.

The COVARIANCE of two random variables X and Y is

$$\mathrm{Cov}(X,Y) \equiv \overline{(X-\bar{X})(Y-\bar{Y})} = \overline{XY} - \bar{X}\bar{Y}$$

2.4 Limit theorems

Let $\{X_i\}$ be a set of iid (independently identically distributed) variables, with $i = 1,\ldots,n$. Define the "sample mean"

$$W_n = \sum_{i=1}^{n} X_i/n.$$

Then:

(i) $P[|W_n - \bar{X}| \geq x] \leq \sigma_X^2/(nx^2)$

(ii) $\lim_{n\to\infty} W_n = \bar{X}$ with probability one (strong law of large numbers)

(iii) $\lim_{n\to\infty} P[Z_n < x] = \Phi(x)$ (CENTRAL LIMIT THEOREM)

where $Z_n \equiv (1/\sigma_X\sqrt{n})(\sum_{i=1}^{n} X_i - n\bar{X})$; $\Phi(x) \equiv \int_{-\infty}^{\infty} \exp(-y^2/2)dy\ /\sqrt{2\pi}$

$\Phi(x)$ is the NORMAL DISTRIBUTION.

2.5 The role of transforms

2.5.1 Introduction

In the chapters on Markov processes it will be shown that the probability distributions for our models are the solutions of linear difference or differential equations with constant coefficients. The standard method for solving such equations is to use z- or Laplace transforms respectively.

2.5.2 The z-transform

(a) Let f_n be a function of discrete time, where n=0,1,2,... . The z-transform $F(z)$ of f_n is defined by

$$F(z) \equiv \sum_{n=0}^{\infty} f_n z^n, \text{ where } z \text{ is complex.}$$

This relationship is written as $f_n \Leftrightarrow F(z)$. The transform exists if there is an $a>0$ such that $\lim_{n\to\infty} |f_n|/a^n = 0$.

The CONVOLUTION PROPERTY of the z-transform is $f_n \circledast g_n \Leftrightarrow G(z)F(z)$, where

$$f_n \circledast g_n = \sum_{k=0}^{\infty} f_{n-k} g_k$$

Given a z-transform $F(z)$, we may recover the sequence f_n by two methods:

$$f_n = (1/n!)\, d^n F(z)/dz^n \big|_{z=0}$$

$$f_n = (-2\pi j)^{-1} \oint_C F(z) z^{-1-n} dz$$

In the second method (the INVERSION FORMULA), the closed contour C must surround all poles of $F(z)$; the Cauchy Residue Theorem may then be applied.

(b) To solve the general difference equation

$$a_N g_{n-N} + a_{N-1} g_{n-N+1} + \dots + a_0 g_n = e_n, \qquad n = k, k+1, \dots$$

where the a_i are known coefficients, e_n is a given function of n, and N boundary equations are given, we proceed as follows. Multiply through by z and sum from k to ∞, i.e.

$$\sum_{n=k}^{\infty} \sum_{i=0}^{n} a_i g_{n-i} z^n = \sum_{n=k}^{\infty} e_n z^n$$

Then pick out expressions for G(z) in this equation and make use of the boundary conditions to eliminate any unknowns. G(z) may then be obtained and inverted.

(c) If X is a random variable and $g_k = P[X=k]$, then G(z) is known as the probability generating function. Note that

$$G(z) = E[z^X] \; ; \quad \partial G/\partial z|_{z=1} = \overline{X} \; ; \quad \partial^2 G/\partial z^2|_{z=1} = \overline{X^2} - \overline{X}$$

2.5.3 The Laplace transform

(a) This is for functions of continuous time. It is defined by

$$F^*(s) \equiv \int_0^{\infty} f(t)\exp(-st)dt \,, \qquad \text{where s is complex.}$$

We write this as $f(t) \Leftrightarrow F^*(s)$. It possesses the convolution property $f(t) \circledast g(t) = F^*(s)G^*(s)$, where the convolution now involves integrals rather than sums. Combining this with the result of Section 2.2, we have an easy way of obtaining the Laplace transform of the pdf of the sum of two independent variables.

A Laplace transform may be inverted by standard techniques such as the inversion integral method.

(b) To solve the general constant-coefficient Nth order linear differential equation

$$a_N d^N f(t)/dt^N + \dots + a_1 df(t)/dt + a_0 f(t) = e(t)$$

we transform both sides to obtain an algebraic equation for $F^*(s)$, which can then be inverted.

(c) The Laplace transform $A^*(s)$ of the pdf of a random variable X may be written as $A^*(s) = E[\exp(-sX)]$. The moments of X may be found by solving

$$d^n A^*(s)/ds^n| = (-1)^n \overline{X^n}$$

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3. STOCHASTIC PROCESSES

3.1 Definition of a stochastic process

A stochastic process is a function $X(t,w) = X(t)$ whose values are random variables. It can be thought of as a family of random variables "indexed" by the time t. Both X and t may be continuous or discrete. The relationships of the X(t) to each other are described by the joint PDF;

$$F_X(x;t) = P[X(t_1) \leq x_1, \ldots\ldots, X(t_n) \leq x_n] \quad \text{for all } x,\ t,\ n.$$

X(t) is said to be STATIONARY if it is invariant to time shifts, i.e.

$$F_X(x;t+\tau) = F_X(x;t)\ ,$$

where $t+\tau$ implies the vector $(t_1+\tau, t_2+\tau, \ldots\ldots)$.

The pdf for a stochastic process X(t) is $f_X(x;t) = \partial F(x;t)/\partial x$.
Then:

$$\overline{X(t)} = \int_{-\infty}^{\infty} x\, f_X(x;t)\, dx\ .$$

3.2 Basic queueing theory

The arrival of customers to a service facility is an example of a stochastic process. The queueing process is characterised by

$$\text{Interarrival time PDF} = A(t) = P[\text{time between arrivals} \leq t]$$

$$\text{Service time PDF} = B(t) = P[\text{service time} \leq t]$$

A queueing process is conventionally written as A/B/m/K, where A and B are the interarrival and service time PDFs respectively, m is the number of servers, and K is the storage capacity.

Let λ be the average arrival rate of customers to the system.

If T is the average time spent in the system by any customer, then the average occupancy N of the system is given by LITTLE'S LAW:

$$N = \lambda T$$

An intuitive "proof" of this law is that an arriving customer should find the same number of customers on average in the system as he leaves behind on his departure.

Let μ be the average service rate. We define the UTILISATION FACTOR ρ as ρ = (average arrival rate)x(average service time) = λ/μ . The system is stable only if $0 \leq \rho < 1$; if $\rho \geq 1$, the number of customers in the system will grow without bound. For a single-server queue, ρ is also the TRAFFIC INTENSITY.

Consider a single-server system and let τ be an arbitrarily long time interval. Then the number of arrivals in this interval will be approximately equal to $\lambda\tau$, by the law of large numbers. If p_0 is the probability that the server is idle at some randomly selected time, then

number of customers served during $\tau \approx \tau(1-p_0)\mu$ with probability 1

$$\therefore \lambda\tau \approx \tau(1-p_0)\mu \qquad \therefore \rho = \lambda/\mu \rightarrow 1-p_0 \text{ as } \tau \rightarrow \infty.$$

$$\therefore \rho = 1-p_0 = \text{fraction of time that server is busy}$$

The above results are true for general interarrival and service time PDFs; in this case the single-server queue is denoted as G/G/1.

In order to arrive at tractable models for communications networks, restrictions must be imposed upon the form of the interarrival and service time PDFs. A common system employed is the M/M/1 queueing system, where M represents the Markovian (or exponential) distribution. The M/M/1 queue is described later in this chapter.

3.3 Some stochastic processes

Stochastic processes become more interesting (and certainly easier to analyse!) if some structure is imposed upon the joint PDF.

A MARKOV PROCESS is a random process in which the probability of the next state being of a particular type depends only on the current state and not upon the system's past history. If the Markov process has a discrete state space, the process is called a MARKOV CHAIN. We shall be interested mostly in Markov chains; the case of a continuous state space is the province of the diffusion approximation [3].

State transitions may occur at any time for a continuous-time Markov chain. The Markov property dictates that the time for which a state may remain unchanged is distributed exponentially. In the case of a discrete-time Markov chain, transitions must be made at every unit time; the Markov property then dictates that the time for which a state may remain unchanged is distributed geometrically.

The BIRTH-DEATH PROCESS is a special case of a Markov chain where transitions may only take place to the current state or neighbouring states.

A SEMI-MARKOV PROCESS is a generalised Markov process in which the times between state changes obey an arbitrary probability distribution (discrete or continuous). A Markov process can still be defined at the instants of transition and is known as an imbedded Markov process.

A RANDOM WALK is an example of a semi-Markov process in which we consider the motion of a particle in a state space such that the next position of the particle is equal to the sum of the previous position and a random variable drawn independently from an arbitrary distribution. This distribution does not depend on the state of the process except perhaps at some boundary states; i.e. a random walk is a sequence $\{S_n;\ n = 1,2,...\}$ which satisfies

$$S_n = X_1 + X_2 + \ldots\ldots + X_n \qquad (n = 1,2,\ldots.)$$

where $S_0 = 0$ and the X_i are independently drawn from the same distribution. The label n counts the number of transitions made. The time intervals between transitions are of no interest in a random walk; what is of interest is the position S_n after n transitions.

A RENEWAL PROCESS is similar to a random walk, but in this case the quantity of interest is the distribution of time between transitions. Thus S_n is now the time at which the nth transition takes place, and X_n is the time between the (n-1)th and nth transitions.

3.4 Discrete-time Markov chains

3.4.1 Introduction

The Markov property for a Markov chain $\{X_n\}$ is

$$P[X(t_{n+1})=x_{n+1} | X(t_n)=x_n,\ldots\ldots,X(t_1)=x_1] \;=\; P[X(t_{n+1})=x_{n+1} | X(t_n)=x_n]$$

where the rhs is the (one step) transition probability. Given the initial distribution $P[X_0 = j]$ and the transition probabilities, we may determine the probability of being in various states at time n.

The Markov chain is HOMOGENEOUS if the transition probabilities are independent of time. We may then define the m-step transition probability

$$p_{ij}^{(m)} \;=\; P[X_{n+m} = j | X_n = i] \;, \quad \text{(n arbitrary and m=1,2,...)}$$

3.4.2 Homogeneous Markov chains

A Markov chain is IRREDUCIBLE if every state can be reached from every other state. Define

$$f_j^{(n)} = P[\text{first return to } E_j \text{ occurs n steps after leaving } E_j]$$

$$\therefore \; P[\text{ever returning to } E_j] \;\equiv\; f_j \;=\; \sum_{n=1}^{\infty} f_j^{(n)}$$

State E_j is RECURRENT if $f_j = 1$ and TRANSIENT if $f_j < 1$. If it is only possible to return to E_j after an integral multiple of γ steps, then E is PERIODIC with period γ ; if $\gamma = 1$, then E_j is APERIODIC.

For recurrent states E_j, we define the mean recurrence time of E_j as

$$M_j \;\equiv\; \sum_{n=1}^{\infty} n \, f_j^{(n)}$$

If $M_j = \infty$, E_j is RECURRENT NULL; if $M_j < \infty$, E_j is RECURRENT NONNULL.

We now introduce two important theorems. Let $\pi_j^{(n)}$ be the probability of finding the system in state E_j at the nth step, i.e.

$$\pi_j^{(n)} \equiv \; P[X_n = j]$$

Then:

Theorem 1 - The states of an irreducible Markov chain are either all transient or all recurrent nonnull or all recurrent null. If periodic, then all states have the same period γ.

A probability distribution P_j is STATIONARY if

$$\pi_j^{(0)} = p_j \Rightarrow \pi_j^{(n)} = p_j \quad \text{for all n.}$$

Theorem 2 - In an irreducible and aperiodic homogeneous Markov chain, the limiting probabilities (equilibrium probabilities)

$$\pi_j = \lim_{n\to\infty} \pi_j^{(n)}$$

always exist and are independent of the initial distribution. Moreover, either

(a) All states are transient or all states are recurrent null, in which cases $\pi_j = 0$ for all j and no stationary distribution exists, or

(b) all states are recurrent nonnull and then $\pi_j > 0$ for all j, in which case the set $\{\pi_j\}$ is a stationary distribution and

$$\pi_j = 1/M_j$$

In this case the π_j are uniquely determined by

$$\sum_i \pi_i = 1 \quad ; \quad \pi_j = \sum_i \pi_i p_{ij}$$

A state E_j is ERGODIC if it is aperiodic, recurrent and nonnull; i.e. $f_j = 1$, $M_j < \infty$, $\gamma = 1$. If all states of a Markov chain are ergodic, then the chain itself is ergodic. Thus a Markov chain is ergodic if any of the following apply:

(i) $\lim_{n\to\infty} \{\pi_j^{(n)}\} = \{\pi_j\}$

(ii) the chain is finite, aperiodic and irreducible

(iii) the set of linear equations in theorem 2 has a nonnull solution for which

$$\sum_j |\pi_j| < \infty$$

Define the probability vector $\pi \equiv [\pi_0, \pi_1, \pi_2, \ldots]$. Then we may write the theorem 2 equations as

$$\pi = \pi P \quad ; \quad \sum_i \pi_i = 1 \qquad (3.1)$$

where $P = [p_{ij}]$ is the transition probability matrix. There is a linear dependence in the set of simultaneous equations for π, so the probability conservation condition in eq.(3.1) is needed to determine the complete solution.

We now consider the TRANSIENT distribution $\pi^{(n)} \equiv [\pi_0^{(n)}, \pi_1^{(n)} \ldots]$. We have

$$\pi^{(n)} = \pi^{(n-1)} P = \pi^{(0)} P^n \quad (n = 1,2,\ldots) \qquad (3.2)$$

3.4.3 Inhomogeneous Markov chains

The results for a homogeneous chain may be generalised to the case of an inhomogeneous chain, where the transition probabilities depend on time. Define

$$p_{ij}(m,n) \equiv P[X_n = j \mid X_m = i]$$

which is the probability that the system will be in state E_j at step n, given that it was in state E_i at step m. In going from step m to n, the system must pass through some intermediate state E_k at some intermediate time q. Thus

$$p_{ij}(m,n) = \sum_k P[X_n=j, X_q=k \mid X_m=i]$$
$$= \sum_k P[X_q=k \mid X_m=i]\, P[X_n=j \mid X_m=i, X_q=k]$$

by definition of conditional probability. Using the Markov property, we obtain the CHAPMAN-KOLMOGOROV (CK) EQUATION

$$p_{ij}(m,n) = \sum_k p_{ik}(m,q)\, p_{kj}(q,n) \qquad (m \leqslant q \leqslant n) \tag{3.3}$$

If we define the matrices $P(n) \equiv [p_{ij}(n,n+1)]$ and $H(m,n) = [p_{ij}(m,n)]$, the CK equation becomes

$$H(m,n) = H(m,q)H(q,n)$$

The FORWARD CK equation is obtained by setting $q=n-1$:

$$H(m,n) = H(m,n-1)P(n-1) \tag{3.4}$$

The solution is

$$H(m,n) = P(m)P(m+1)\ldots\ldots P(n-1) \tag{3.5}$$

As an analogy to the homogeneous case, we have

$\pi^{(n+1)} = \pi^{(n)}P(n)$ with solution $\pi^{(n+1)} = \pi^{(0)}P(0)P(1)\ldots.P(n)$.

3.5 Continuous-time Markov chains

The forward CK equation in this case is

$$\partial p_{ij}(s,t)/\partial t = q_{jj}(t)p_{ij}(s,t) + \sum_{k \neq j} q_{kj}(t)p_{ik}(s,t) \tag{3.6}$$

where q_{ij} is the rate at which the system moves from state E_i to E_j, given that it's currently in E_i. We define

$$P(t) = [p_{ij}(t,t+\Delta t)]$$
$$Q(t) = \lim_{\Delta t \to 0} (P(t)-I)/\Delta t \qquad \text{(the transition rate matrix)}$$
$$H(s,t) = [p_{ij}(s,t)]$$

We may write the forward CK equation as $\partial H(s,t)/\partial t = H(s,t)Q(t)$, $s \leqslant t$. The CK equation for the equilibrium probabilities is

$$d\pi_j(t)/dt = q_{jj}(t)\pi_j(t) + \sum_{k \neq j} q_{kj}(t)\pi_k(t) \tag{3.7}$$

Eq(3.6) describes the probability of the system being in state E_j at time t, given that it was in E_i at time s. Eq(3.7) merely gives the probability that the system is in state E_j at time t; information about the initial state is contained in $\pi(0)$.

For the homogeneous case, we have $Q(t) \to Q$, $H(s,s+t) \to H(t)$. The limiting distribution is given by

$$\pi Q = 0 \quad ; \quad \sum_j \pi_j = 1 \tag{3.8}$$

3.6 Birth-death processes

3.6.1 The Chapman-Kolmogorov equations

Elementary queueing theory is built upon the theory of birth-death processes, which is the special case of a Markov process in which only the transitions $E_k \to E_{k-1}$ and $E_k \to E_{k+1}$ are allowed. We shall now derive some properties of a birth-death process in the language of queueing theory. Consider a queueing system which increases or decreases its population by means of arrivals and departures respectively. Define

λ_k = arrival rate when system contains k customers

μ_k = service " " " " "

In terms of the transition rate matrix Q, we have

$$q_{k,k+1} = \lambda_k \; ; \qquad q_{k,k-1} = \mu_k$$

Since $\sum_j p_{ij}(s,t) = 1$, then by definition of Q we have $\sum_j q_{ij}(t) = 0$.

This implies that $q_{kk} = -(\mu_k + \lambda_k)$

Finally, $q_{jk} = 0$ for $|j-k|>1$ by definition of the birth-death process.

We assume that the queueing process is a homogeneous continuous-time Markov process on the discrete states 0,1,2,...., that arrivals and departures are independent(i.e. the Markov property), and

$$\begin{aligned} &P[\text{exactly one arrival in } (t,t+\Delta t)\,|\,k \text{ in system}] = \lambda_k \Delta t + o(\Delta t) \\ &P[\text{exactly no arrivals in } (t,t+\Delta t)\,|\,k \text{ in system}] = 1 - \lambda_k \Delta t + o(\Delta t) \end{aligned} \qquad (3.9)$$

We obtain two more equations by substituting "departure" for "arrival" and μ_k for λ_k. Note that we have assumed that the probability of multiple events is $o(\Delta t)$. Define

$P_k(t)$ = probability that system contains k customers at time t

Applying eq(3.7) with $\pi_k(t) = P_k(t)$, we have

$$\begin{aligned} dP_k(t)/dt &= -(\lambda_k+\mu_k)P_k(t) + \lambda_{k-1}P_{k-1}(t) + \mu_{k+1}P_{k+1}(t)\,, \quad k \geq 1 \\ dP_0(t)/dt &= -\lambda_0 P_0(t) + \mu_1 P_1(t)\,, \quad k = 0 \end{aligned} \qquad (3.10)$$

A simpler method of obtaining eq(3.10) is to consider the state-transition-rate diagram (Fig. 1). The equation

(Effective probability flow rate into E_k)

= (Flow rate into E_k) - (Flow rate out of E_k)

immediately yields eq(3.10). This is an example of GLOBAL BALANCE, about which more will be said later.

3.6.2 Poisson arrivals

Consider a pure arrival system with $\lambda_i = \lambda$, $\mu_i = 0$. Then the solution of eq(3.10) is

$$P_k(t) = (\lambda t)^k \exp(-\lambda t) / k! ,$$

i.e the Poisson distribution. The mean and variance of this distribution are both equal to λt, and $G(z) = \exp[\lambda t(z-1)]$.

Let $\tilde{t}$ be the random variable describing interarrival times. Then:

$$A(t) = 1 - P[\hat{t} > t] = 1 - P_0(t) = 1 - \exp(-\lambda t) , \quad t > 0$$

$$a(t) = dA(t)/dt = \lambda \exp(-\lambda t) , \quad t \geqslant 0$$

This is the exponential distribution for interarrival times. We have

$$E[\hat{t}] = 1/\lambda \quad ; \quad E[(\tilde{t})^2] = 2/\lambda^2 \quad ; \quad \sigma_{\tilde{t}}^2 = 1/\lambda^2$$

3.6.3 Postscript

It is a sad fact that the solution of eq(3.10) for even the simplest interesting queueing system (i.e. M/M/1, with $\lambda_i = \lambda$, $\mu_i = \mu$) is far too complicated to be useful [2]. The diffusion approximation is therefore preferable for examining dynamic behaviour, although some attempts have been made to apply queueing theory to the transient behaviour of the M/M/1/K queue [9]. The power of queueing theory lies in its ability to describe equilibrium distributions.

3.7 Queueing systems in equilibrium

We now obtain the general equilibrium distribution for a single queue. Let

$$p_k = \lim_{t \to \infty} P_k(t) , \qquad \lim_{t \to \infty} dP_k(t)/dt = 0$$

where $P_k(t)$ is as defined in eq(3.10). Setting $\lambda_{-i} = \mu_{-i+1} = p_{-i} = 0$ $(i=1,2,\ldots)$ in order to avoid writing the k=0 boundary equation separately, we obtain from eq(3.10)

$$p_k = p_0 \sum_{i=0}^{k-1} \lambda_i/\mu_{i+1} , \qquad k \geqslant 0$$

$$p_0 = \left[1 + \sum_{k=1}^{\infty} \prod_{i=0}^{k-1} \lambda_i/\mu_{i+1} \right]^{-1} \quad \text{(from conservation of probability)} \tag{3.11}$$

We are considering equilibrium distributions and thus are interested only in the ergodic case; this occurs if there is a $k_0 \geqslant 0$ such that $\lambda_k/\mu_k < 1$ for all $k \geqslant k_0$.

We now consider some specific queueing systems. Define $\rho \equiv \lambda/\mu$, N as the average occupancy of the system, and T as the average system delay. M and G represent the Markovian and general distributions respectively.

(a) The M/M/1 queue
Choose $\lambda_k = \lambda$, $\mu_k = \mu$. For ergodicity, $\rho < 1$. We have

$$p_k = (1-\rho)\rho^k \; ; \quad N = \rho/(1-\rho) \quad ; \quad T = 1/(\mu - \lambda)$$

(b) The M/M/m queue
Choose $\lambda_k = \mu$, $\mu_k = k\mu$ $(0 \leqslant k \leqslant m)$ or $m\mu$ $(m \leqslant k)$

For ergodicity, $\lambda/m\mu < 1$. We obtain Erlang's C formula for the probability that an arriving customer must join a queue, i.e.

$$P[\text{queueing}] = \sum_{k=m}^{\infty} p_k \equiv C(m, \lambda/\mu)$$

(c) The M/M/1/K queue
We must "lose" any customers which arrive when the system contains K customers. Choose $\lambda_k = \lambda$ (k < K) or 0 (k $\geqslant$ K) and $\mu_k = \mu$.
We obtain

$$p_k = [(1-\rho)/(1-\rho^{K+1})]\rho^k \quad , \quad 0 \leqslant k \leqslant K$$

(d) The M/G/1 queue
Although the analysis of a queue with general service-time distribution requires techniques we have not yet covered, it is convenient here to give the POLLACZEK-KHINCHIN FORMULA for the average occupancy of an M/G/1 queue:

$$N = \rho + \rho^2(1+C_b^2)/2/(1-\rho) \quad ,$$

where $C_b^2 = \sigma_b^2/\mu^2$, where σ_b^2 is the variance of the service time.

3.8 Residual life

The queueing systems M/G/m and G/M/m are more difficult to analyse. Consider the M/G/1 system for example: as the departures do not now constitute a "memoryless" process, the state description must include the time since the most recent departure as well as the number of customers in the system. We can consider the set of departure times as defining an IMBEDDED Markov chain, where now the transition probabilities are not merely of the birth-death type since many arrivals may occur during a departure period. We must deal with the case of an arriving customer finding a partially served customer in the service facility. We thus introduce the important concept of RESIDUAL LIFE. Consider an arbitrary interdeparture time PDF

$$F(x) = P[\tau_{k+1} - \tau_k \leqslant x] \quad ,$$

where τ_k is the departure time of the kth customer. Then the pdf is given by $f(x) = dF(x)/dx$. Take a randomly chosen instant, t (Fig. 2). The time Y until the next departure is the residual life, with PDF $\hat{F}(x) = P[Y \leqslant x]$ and pdf $\hat{f}(x)$. Let the selected lifetime X have PDF $F_X(x)$ and pdf $f_X(x)$. Then

(Probability that interval of length x is chosen)

$\propto$ (length of interval) x (relative occurrence of interval)

i.e. $P[x<X\leqslant x+dx] = f_X(x)dx = x\, f(x)\, dx / m_1$

where $1/m_1$ normalises the probability and $m_n \equiv E[(\tau_k - \tau_{k-1})^n]$.
Thus:

$$f_X(x) = x\, f(x) / m_1 \tag{3.12}$$

Now it's obvious that $P[Y\leqslant y | X=x] = y/x$ for $0 \leqslant y \leqslant x$.

$\therefore\ P[y<Y\leqslant y+dy,\ x<X\leqslant x+dx] = (dy/x)\,(xf(x)dx/m_1) = f(x)dydx/m_1$

Integrating wrt x:

$$f_Y(y) = (1-F(y)) / m_1 \qquad (3.13)$$

We may use Laplace transforms to obtain the moments of this distribution. In particular:

$$E[Y] = m_2 / 2m_1 \qquad (3.14)$$

3.9 Networks of Markovian queues

3.9.1 Burke's output theorem

Our results so far apply to single queues. The question arises as to what relevance queueing theory has to a NETWORK of queues, in which the output of one queue becomes the input of another.

We consider an M/M/1 queueing system with Poisson arrivals at rate λ and an exponential server of rate μ (Fig. 3). We wish to find the PDF D(t) of the interdeparture time distribution which feeds the next node in the network. Let $A^*(s)$, $D^*(s)$ and $B^*(s)$ be the Laplace transforms of the interarrival, interdeparture and service time pdf's respectively. When a customer leaves the node, either a second customer is in the queue or the queue is empty. In the first case, D(t) is equal to B(t), so

$$D^*(s)|\text{node nonempty} = B^*(s)$$

If the node is empty after the first customer departs, then

(total delay) = (time until 2nd customer arrives)
+ (service time for 2nd customer)

As the variables are independent and identically distributed, the Laplace transform of the sum is equal to the product of the Laplace transforms of the individual times (see sections 2.2 and 2.5);

$$D^*(s)|\text{node empty} = A^*(s)B^*(s) = [\lambda/(s+\lambda)][\mu/(s+\mu)]$$

For exponential pdf's, the probability of a departure leaving behind an empty system is the same as the probability of an arrival finding an empty system, i.e. $1-\rho$, where $\rho = \lambda/\mu$. Thus:

$$D^*(s) = (1-\rho)D^*(s)|\text{node empty} + \rho D^*(s)|\text{node nonempty} = \lambda/(s+\lambda)$$

$$D(t) = 1 - \exp(-\lambda t) = A(t)$$

Thus the Poisson stream is in no way affected by its passage through the node.

It can also be shown that the split and join of Poisson streams are themselves Poisson. These are important results, as they imply that the individual queues in a network of M/M/1 queues may be analysed independently of one another. This decomposition is essential for a tractable analytic model.

3.9.2 Open networks

Consider an arbitrary open network of queues with N nodes. The ith node consists of m exponential servers each with parameter μ_i, and receives Poisson arrivals from outside the system at rate γ_i. Customers move from node i to node j with probability r_{ij} or leave the system with probability

$$1 - \sum_{j=1}^{N} r_{ij} \; .$$

Feedback (i.e. $r_{ii}>0$) is allowed.

The total arrival rate λ_i to node i is the sum of the Poisson external arrivals and the internal arrivals (which are not necessarily Poisson because of feedback), i.e.

$$\lambda_i = \gamma_i + \sum_{j=1}^{N} \lambda_j r_{ji} \qquad (i = 1,2,\ldots\ldots,N) \qquad (3.15)$$

or, in matrix form, $\lambda = \gamma + \lambda R$, where $\lambda \equiv [\lambda_1, \lambda_2, \ldots\ldots, \lambda_N]$. Each node must satisfy $\lambda_i < m_i \mu_i$ for ergodicity.

Jackson [10] showed that each node i behaves as if it were an M/M/1 system with a Poisson input rate λ_i, even though the inputs are not Poisson in general. Let k_i be the number of customers in node i. Then the equilibrium probability distribution factors into the product of the marginal distributions, i.e.

$$p(k_1,k_2,\ldots\ldots,k_N) = p_1(k_1)p_2(k_2)\ldots\ldots p_N(k_N) \; ,$$

where each marginal distribution is the solution to the M/M/m system. This result is known as JACKSON'S THEOREM.

3.9.3 Closed networks

A closed Markovian network contains a fixed number K of customers and no external arrivals or departures are permitted. This constraint introduces a dependency among the elements k_i of the state vector, as they must sum to K. By considering the balancing of probability flows (section 3.6), Gordon and Newell [11] proved that

$$p(k_1,k_2,\ldots\ldots,k_N) = [G(K)]^{-1} \prod_{i=1}^{N} \rho_i^{k_i} / \beta_i(k_i) \qquad (3.16)$$

where

(i) $G(K) = \sum_{k \in A} \prod_{i=1}^{N} \rho_i^{k_i} / \beta_i(k_i)$

(ii) ρ_i are defined by $\rho_i = \lambda_i/\mu_i$, where the λ_i are the solutions to $\lambda = \lambda R$ (to within a multiplicative constant).

(iii) $\beta_i(k_i) = k_i!$ $(k_i < m_i)$ or $m_i!\, m_i^{k_i - m_i}$ $(k_i \geq m_i)$

(iv) A is the set of state vectors k for which $\sum_{i=1}^{N} k_i = K$.

Consider the quantities ρ_i / m_i, and suppose that there exists a largest ratio ρ_k/m_k, say. Then it can be shown that an infinite number of customers will form in node k, which is the "bottleneck" for the network, as $K \to \infty$. In this limit, a product-form solution exists for the marginal distribution

$$p(k_1,k_2,\ldots\ldots,k_N) = p_1(k_1)p_2(k_2)\ldots\ldots p_N(k_N)$$

3.10 Global and local balance

The most general model to date of a product-form queueing network is that of Baskett et al [12]. They generalise the models of the previous Section to include different types of server and multiple customer classes. The four types of service centre can model central processors, data channels, terminals and routing delays respectively.

To obtain the joint equilibrium distribution, we must solve the forward CK equation with dP/dt=0, or, equivalently, solve the global balance equation

$$\sum_{\text{all states } E_j} P(E_j)\,[\text{rate of flow from } E_j \text{ to } E_i] = P(E_i)\,[\text{rate of flow out of } E_i] \quad (3.17)$$

for all states E_i. With a complex state description such as that in [12], the global balance equations become difficult to solve. We can obtain sufficient conditions for global balance by decomposing the global balance equations into a larger set of smaller LOCAL BALANCE equations. For each state E_i and node k, we equate the rate of flow into E_i by a customer entering node k to the rate of flow out of E_i by a customer leaving node k. Thus each global balance equation is a sum of local balance equations. The local balance method is used to determine the joint equilibrium distribution in [12]: a discussion of local balance is given in [13].

Fig. 1 State transition diagram (Section 3.6.1)

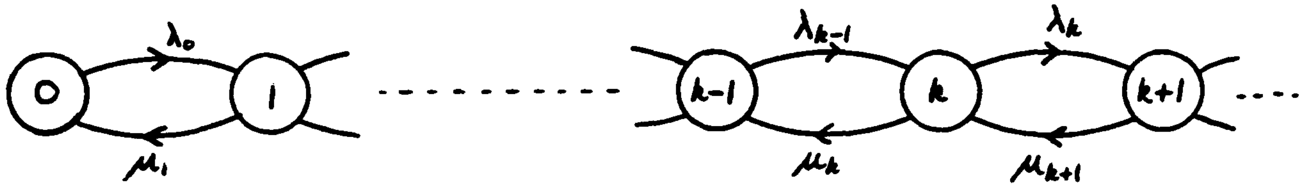


Fig. 2 Residual life (Section 3.8)

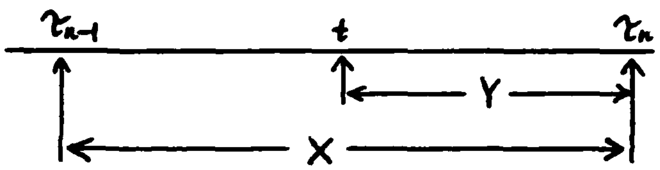
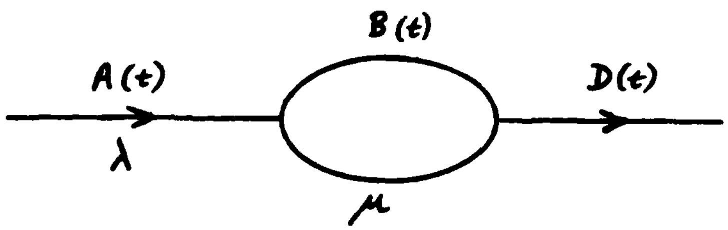


Fig. 3 The output theorem (Section 3.9.1)



4. APPLICATION TO COMMUNICATIONS NETWORKS

4.1 Introduction

4.1.1 Real networks

A communications network consists of a subnet (containing data channels, packet buffers and nodal processors) and hosts which feed packets into the subnet. Because of resource sharing, queues may build up for the data channels. The progress of packets in the network may be governed by a number of protocols.

Preallocation of resources (e.g. circuit switching) is a safe but wasteful method of sending messages. Therefore in a store-and-forward network, only a channel and a buffer at each end of the channel are allocated for each packet. If the packet is successfully received at the next node, an acknowledgement (ack) is sent to the previous node. If no ack has been received at the end of a timeout period, the packet is retransmitted.

The datagram and virtual circuit subnet services are now introduced. The datagram traverses the network as an independent entity; virtual circuit packets belong to a "virtual channel" connecting sources and sinks and are usually characterised by the same routing behaviour.

The efficient utilisation of resources depends upon the routing algorithm and flow and congestion control. Performance may be defined by average throughput and transit delay.

4.1.2 Assumptions

In order to apply the theory of product-form queueing networks [14,15], certain assumptions and approximations must be made. Adaptive routing, message priorities and loss of packets caused by full buffers cannot be treated because of statistical dependence between the elements of the state vector.

Most models of communication networks assume exponential arrivals and departures. Additionally, Kleinrock's INDEPENDENCE ASSUMPTION [3] must be invoked: every time a packet joins a queue, its length must be redetermined from the exponential pdf associated with the service facility. This assumption removes the statistical dependence of transmission times at the channels. It is not a realistic approximation, but this cannot be helped.

If the average arrival rate of packets to a node is λ, then the traffic intensity is given by $\rho = \lambda/\mu C$, where C is the line capacity of the next channel and μ is the reciprocal mean packet length.

4.1.3 Mean end-to-end delay in an open network [3]

Let γ be the total average traffic rate offered to the network. This is split up among the various channels such that λ_i is the average rate to channel i (which has capacity C_i). We may then define a traffic intensity $\rho_i = \lambda_i/\mu C_i$ at each channel. Assume there are M channels in the network. Then the mean number of packets in the network is equal to the sum of the mean queue lengths $\bar{n}_i$ at each channel; if T is the mean end-to-end delay for an arbitrary packet, then by Little's law the mean total number of packets is γT. Thus

$$\gamma T = \sum_{i=1}^{M} \overline{n_i} = \sum_{i=1}^{M} \rho_i/(1-\rho_i) = \sum_{i=1}^{M} \lambda_i/(\mu C_i - \lambda_i)$$

$$\therefore \quad T = \gamma^{-1} \sum_{i=1}^{M} \lambda_i/(\mu C_i - \lambda_i) \tag{4.1}$$

We may optimise line capacities by minimising T wrt the C_i, subject to some cost constraint. If the constraint is linear in the C_i, then the method of Lagrange multipliers may be used.

The full distribution for end-to-end delay along a path has also been evaluated [16]. In particular, the variance of end-to-end delay along the path is

$$\sigma_{path}^{2} = \sum_{i \in path} [\mu C_i(1-\rho_i)]^{-2} \tag{4.2}$$

4.1.4 Closed networks

Closed networks with product form may be used to model end-to-end flow control in a virtual circuit and permit-oriented (isarithmic) global congestion control. The evaluation of the PARTITION FUNCTION (or normalisation constant) G(K) in eq(3.16) is a problem because of the large computer time and storage required when there is a large number of nodes. Another difficulty is that the ρ_i are determined only to within a multiplicative constant, so that incorrect scaling may cause underflow or overflow.

There are currently three approaches to the calculation of the properties of a closed network: the convolution algorithm [17], the mean-value-analysis (MVA) algorithm [18], and the integral representation and asymptotic expansion of the partition function [19].

MVA obtains mean quantities associated with a closed chain without having to deal with the full product-form expression for the equilibrium distribution. Consider a closed cyclic chain of N exponential servers with mean service times τ_n (n = 1,2,...,N) and a fixed number K of circulating messages. Define

$\overline{k_n}(K)$ = mean size of queue n (including message in service)

$\overline{t_n}(K)$ = mean delay at queue n

$\lambda(K)$ = throughput of chain

The mean delay at queue n for a "tagged" message is the sum of the service time τ_n for the tagged message and the service times for the average number of messages in the system when the tagged message arrives. The ARRIVAL THEOREM states that in a closed exponential system the state seen upon arrival instants has the same distribution as the equilibrium distribution of the same closed system with K-1 messages. Thus

$$\overline{t_n}(K) = \tau_n + \tau_n \overline{k_n}(K-1) \qquad (n = 1,2,\ldots,N) \tag{4.3}$$

With the obvious equation $\overline{k_n}(0) = 0$, and the following expressions (obtained by the application of Little's formula to all N servers and to each individual server respectively), we have a simple recursive solution for the closed exponential system:

$$\lambda(K) = K / \sum_{n=1}^{N} \overline{t_n}(K) \tag{4.4}$$

$$\overline{k_n}(K) = \lambda(K)\overline{t_n}(K) \tag{4.5}$$

MVA can be extended to include more general topologies than the cyclic chain [18].

4.1.5 Homogeneous network models

Networks with message priorities or nodal buffer limits contain statistical dependencies which do not allow a product-form solution for the equilibrium distribution. A homogeneous network approximation may be employed, in which all nodes are identical and perform identical functions. This implies that all nodes are topologically equivalent, all channels are of the same capacity, and all external offered traffic rates are equal. Such models have been used to investigate networks with priorities [20] and nodal buffer management schemes [21,22].

4.1.6 The range of application of queueing theory

The model defined by eq(4.2) may be used to optimise non-adaptive routing algorithms. The use of closed product-form networks for the analysis of end-to-end flow control and isarithmic congestion control has also been mentioned in this section.

We may investigate network components such as switches and point-to-point links by applying directly local or global balance conditions; the performance of buffer management schemes and data-link-control (DLC) protocols may thus be evaluated.

4.2 Data-link-control protocols

4.2.1 A simple model with window flow control

Butto et al [23] consider two nodes A, B, where B acknowledges packets sent by A. The timeout is represented by a feedback to A of rate θ ; when the timeout expires, all packets in queue Q are erased from Q and placed in Q for retransmission (Fig. 4). The number of packets in Q may not exceed the window size, M.

We assume an exponential arrival pdf with parameter λ , and exponential servers at nodes A and B with parameters α and β respectively. The model is inspired by Ref 24.

The state transition diagrams are given in Fig. 5. The balance equations may be solved by applying the z-transform

$$Q_j(z) = \sum_{i=0}^{\infty} P_{ij} z^i \quad (|z| < 1, \ 0 \leqslant j \leqslant M)$$

Once the probabilities P_{ij} have been obtained, it is easy to calculate the average packet delay by Little's law:

$$\text{Average delay} = \lambda_0^{-1} \sum_{i=0}^{\infty} \sum_{j=0}^{M} (i+j) P_{ij}$$

The graph of this quantity (Fig. 6) shows that the delay is sensitive to a certain range of window widths.

4.2.2 Retransmissions and stability [23]

We now investigate the instability that sets in when retransmissions waste the available resources. Assume an exponential distribution for the service time T, and let the timeout duration be T. Then retransmission occurs if

$$\text{Packet delay } W > T_0 \equiv T - W_A ,$$

where W_A is the (constant) delay time for an ack. The average traffic rate λ offered to the channel is related to the original offered traffic rate λ_0 by $\lambda = \lambda_0 N$, where N is the mean number of transmissions for any one packet (Fig. 7). Now,

$$N = \sum_{i=1}^{\infty} iP_i ,$$

where $P_i \equiv P[\text{i copies sent (including original)}]$

$= P[\text{i-1 failures}]\ P[\text{one success}]$

Now, $P[\text{success}] = P[W \leq T_0] \equiv F_W(T_0)$ (i.e. the PDF for W)

$$\therefore\ P_i = [1 - F_W(T_0)]\ F_W(T_0)$$

Thus: $$\lambda = \lambda_0 \sum_{i=1}^{\infty} i[1 - F_W(T_0)]^{i-1} F_W(T_0) = \lambda_0 / F_W(T_0) ,$$

where λ_0 is the throughput in steady-state conditions.

Using the standard queueing theory expressions for $F_W(T_0)$, an implicit equation for mean packet delay $\bar{W}$ as a function of λ_0 may be obtained. The numerical solution yields the graph in Fig. 8, where the instability is clearly seen.

4.2.3 The "send and wait" protocol

We now introduce a more general treatment by Fayolle et al [25] of the send and wait protocol, which is a case of window flow control with the window size set to unity.

Define the random variables X, Y and Z as the message transmission duration (iid for all messages), the delay from the end of transmission to receipt of an ack, and the waiting time for an ack if timeout $T = \infty$, respectively. Let L be the probability that a message is not acknowledged (due to message loss or erroneous transmission). We may define the PDFs

$$G(x) \equiv P[X \leq x] ,\quad B(x) \equiv P[Y \leq x] ,\quad A(x) \equiv P[Z \leq x] = \begin{cases} B(x)(1-L) & (x < \infty) \\ \text{or } 1 & (x = \infty) \end{cases}$$

The total transmission time τ is typically made up as in Fig. 9. Define

$$D = P[\text{ack not received before timeout}] = 1 - A(T) = 1 - B(T)(1-L)$$

If τ_n is a random variable representing the total effective transmission delay (including all the "send" and "wait" intervals) for a message, given that it has to be transmitted n times before the ack precedes the timeout, then

$$E[\tau_n] = nE[X] + (n-1)T + (1-D)^{-1}\int_0^T ta(t)dt \qquad (4.6)$$

where a(t) is the pdf of X.

The terms in the rhs of eq(4.6) represent respectively the average total transmission duration, total timeout duration, and average time before the ack arrives successfully on the nth attempt. Note the renormalising of the pdf in the third term because of the restricted interval (O,T).

The average effective total transmission duration is then given by

$$E[\tau] = \sum_{n=1}^{\infty} E[\tau_n]\, P[n \text{ transmissions}] = \sum_{n=1}^{\infty} E[\tau_n]D^{n-1}(1-D) \qquad (4.7)$$

The timeout which minimises $E[\tau]$ can now be obtained implicitly in terms of $a(x)$ and $E[X]$.

It is instructive to obtain the distribution of τ by calculating the Laplace transform $\hat{f}(s)$ of the pdf $f(x)$ of τ. We have

$$\tau_n = nX + (n-1)T + Z \qquad (0 \leqslant Z < T) \qquad (4.8)$$

Let $\mathcal{L}[Q]$ denote the Laplace transform of the pdf of the random variable Q. Then the convolution property for the sum of independent variables (Section 2.2) yields

$$\mathcal{L}[\tau_n] = \mathcal{L}[nX]\mathcal{L}[(n-1)T]\mathcal{L}[Z] \qquad (0 \leqslant Z < T)$$

Now, $P[nX \leqslant x] = P[X \leqslant x/n] = G(x/n)$

$$\therefore \mathcal{L}[nX] = \int_0^{\infty} \exp(-sx)g(x/n)d(x/n) = \int_0^{\infty} \exp(-sy)g(y)dy = \hat{g}(sn)$$

The above refers to retransmissions of the same message, which are obviously not independent. If the durations X were independent, we would have $\mathcal{L}[nX] = [\hat{g}(s)]^n$.

We also have

$$\mathcal{L}[(n-1)T] = \int_0^{\infty} \exp(-sx)\delta[x-(n-1)T]dx = \exp[-s(n-1)T]$$

and

$$\mathcal{L}[Z] = \hat{a}(s)/(1-D) \qquad (0 \leqslant Z < T)$$

Finally,

$$\mathcal{L}[\tau] = \sum_{n=1}^{\infty} D^{n-1}(1-D)\mathcal{L}[\tau_n] \quad ,$$

which may be differentiated to give the moments of the PDF of τ.

The above analysis is completely general and does not depend upon the specific distributions G(x) and B(x). Fayolle et al then consider the effect of the protocol on the buffer queue behaviour by assuming an exponential arrival process for G(x) and using the imbedded Markov chain approach for the number of messages in the buffer. Results are given [25] for the optimal timeouts under these conditions.

Another approach to the analysis of the send and wait protocol is described by Reiser [8]. Both ends of the link may transmit and receive messages; in order to reduce the number of states to be considered, saturated (nonempty) queues are assumed. The THROUGHPUT-LIMIT THEOREM states that the maximum arrival rate leading to a stationary solution is also the maximum achievable throughput; this implies that the maximum throughput may be determined without having to consider those states containing an empty queue. Because of saturation, no acks may be sent until both ends of the link are ready, and so the acks are synchronised. The process is depicted in Fig. 10. TW represents the state in which a message is being transmitted over channel 1 whilst channel 2 is waiting, etc.

We assume that message transmission rates on channels 1 and 2 are exponentially distributed with parameters μ_1, μ_2 respectively; ack rates on both channels are exponentially distributed with parameter μ_A. The state transition diagram is given in Fig. 10. The flows from state TT deserve explanation. If l_1, l_2 are exponentially distributed transmission rates with parameters μ_1, μ_2 respectively, then the variable $\min(l_1, l_2)$ is also exponentially distributed with parameter $\mu_1 + \mu_2$. Similarly,

$$P[l_1 \leqslant l_2] = \mu_2/(\mu_1 + \mu_2).$$

Thus the flow from TT to TW, for example, is given by

$$(\mu_1 + \mu_2)\, P[l_1 \leqslant l_2] = \mu_2 .$$

The balance equations for the state transitions are easily solved to yield the maximum throughput in terms of μ_1, μ_2 and μ_A. (Alternatively, as the system can be thought of as a closed exponential one with a single message, we may apply the mean value analysis of Section 4.1 with K=1.)

4.2.4 The HDLC Protocol

Bux et al [27] apply a heuristical method to the analysis of the HDLC protocol. The first two moments of the PDF of the effective transmission time are evaluated approximately by making independence assumptions and using the renewal formula (eq. 3.14). The mean transfer time of messages, assuming an exponential arrival distribution at either end of the link, is then given by the Pollaczek-Khinchin formula for the M/G/1 queue (Section 3.7).

Labetoulle et al [28] consider the case of saturated queues only.

4.3 Nodal buffer management

4.3.1 Introduction

Some difficulties arise in the performance evaluation of buffer management strategies because the equilibrium probability distribution for a queueing network with finite storage is unknown at present. The standard approach is to analyse the behaviour of a single node by means of queueing theory and then assume flow conservation (perhaps in a homogeneous network).

4.3.2 Restricted Buffer Sharing

Irland [31] analyses a policy in which R classes of traffic are accepted by R output channel queues. Each class r has exponential arrival and server rates with parameters λ_r, μ_r respectively. If there is a total of N buffers in the node, then the strategy is to limit the number of buffers k_r occupied by packets of class r to $M (\leqslant N)$.

The easiest way to find the equilibrium probabilities is to consider the local balance equations:

$$\mu_r P(k_1,\ldots,k_r+1,\ldots k_R) = \lambda_r P(k_1,\ldots,k_r,\ldots,k_R) \quad \text{for } r=1,2,\ldots,R$$

The solution is $\quad P(\underline{k}) = C^{-1} \prod_{r=1}^{R} \rho_r^{k_r}$ (for feasible k only),

where $\quad C = \sum_{\text{feasible } \underline{k}} \prod_{r=1}^{R} \rho_r^{k_r}$ and $\rho_r = \lambda_r/\mu_r$.

The normalisation constant C is difficult to evaluate because of the restriction on the states; Irland achieves this by a convolution approach. The nodal blocking probability $P[k_r=M]$ for class r packets can then be found by numerical means; the optimal value M^* of M which minimises this blocking probability turns out to be

$$M^* \approx N/\sqrt{R}$$

Irland also finds the throughput vs. load behaviour for various other strategies such as "no sharing", with $M = N/R$, and "unrestricted sharing", with $M=N$. The worst congesion occurs for the unrestricted case, as expected. The behaviour of a network of nodes is not considered.

4.3.3 A simple model with congestion [29]

Before describing other results in this field, it is useful to consider the fundamental model on which they are based. A node with one input and one output link has a finite number N of buffers; there are exponential offered arrival and departure rates with parameters λ, μ. The actual throughput γ is less than λ because of the effects of blocking at the next node. Let the blocking probability be B (Fig. 11).

Assume that the blocking probability is the same for the next node in the network (i.e. a homogeneous network). The effective utilisation ρ' is given by

$$\rho' = \lambda/\mu/(1-B)$$

From queueing theory, $\quad B = (1-\rho')\rho'^N/(1-\rho'^{N+1}) \quad$ (see Section 3.7)

Then the equation $\gamma = \lambda\,(1-B)$ is an implicit one for γ (the throughput) as a function of λ (the offered traffic rate). The solution yields the congestion curve of Fig. 12. It must be noted that flow balancing arguments have been used to obtain a stationary blocking probability, although the system is not stationary [8].

4.3.4 Lam's model of a store-and-forward node [30]

Lam's model is an enhancement of that given above. He constructs a product- form queueing network model of a node, with different classes of traffic routed to several output links with Markovian transition probabilities. Acks and timeouts are represented by random delays (i.e. infinite-server "queues", which support product form). Packets destined for node j are routed to the timeout box with probability B_j or to the ack box with probability $1-B_j$. B_j is thus the blocking probability at node j. The acked packets are sent to the finite buffer pool.

The equilibrium distribution for the node is given by th standard open network solution. We construct a network approximation by assuming that the total flow into any service facility is equal to the flow out of it. (N.B. this is not a homogeneous network assumption, as the blocking probabilities here are not assumed to be the same for each node.) Define the row vectors $\underline{\gamma}$, $\underline{S}$ as the throughput and external offered traffic rate respectively, and let $\underline{P}$ be the node-node routing probability matrix. Then by flow conservation

$$\underline{\gamma} = \underline{S} + \underline{\gamma}\,\underline{P}\,, \quad \text{i.e. } \underline{\gamma} = \underline{S}\,(\underline{I} - \underline{P})^{-1}$$

The actual arrival rate $\underline{\lambda}$ (including retransmissions) is given by an argument similar to that in Section 4.2:

$$\lambda_i = \sum_{j=1}^{M} \gamma_j p_{ji} \,/(1 - B_i)$$

Substituting this in the equation B_i = P[all buffers are filled], we obtain a set of nonlinear simultaneous equations which can be solved numerically (by the Newton-Raphson method, for example). The equations are of the form

$$B_i = f_i(\underline{B})$$

The dependence of B_i on the blocking probabilities $B_j (j \neq i)$ is brought about by the dependence of the normalising constant on the B_j.

4.3.5 Input buffer limiting

Lam and Reiser [21] embed Lam's model of a node in a homogeneous network in order to study the performance of an input buffer limiting mechanism. We shall ignore the nodal processing and acks with timeout and consider only one outgoing link queue in order to concentrate upon the essential characteristics of the buffer limiting method.

Two types of traffic enter the node: input traffic from a local host and transit traffic (i.e. that whic has already passed through one or more nodes) from the rest of the network. The traffic is considered to be exponential with parameters λ_I and λ_T for the input and transit traffic respectively (Fig. 13). Because of blocking caused by the finite storage, the average throughput is given by γ_I and γ_T for input and transit traffic, where

$$\gamma_I = \lambda_I(1 - B_I) \quad , \quad \gamma_T = \lambda_T(1 - B_T)$$

B_I and B_T are the blocking probabilities for the traffic types. In a homogeneous network, these blocking probabilities are the same for all nodes; moreover

$$\gamma_T = \bar{n}\,\gamma_I$$

where $\bar{n}$ is the average number of hops traversed by packets in the network. This equation merely states that transit traffic is generated by input traffic at other nodes.

In order to alleviate congestion, we do not allow input packets to use more than $N_I < N$ buffers, whereas transit packets may use all N buffers. B_I and B_T may be found by an iterative method; using these values, a numerical approach reveals the following rule of thumb for the value of the ratio N_I/N which maximises throughput:

$$N_I/N \leqslant \gamma_I/(\gamma_I + \gamma_T) \quad (= 1/(1+\bar{n}) \text{ for a homogeneous network})$$

Kamoun [22] has considered a slightly different buffer limiting mechanism in which no input traffic is accepted if the total number of filled buffers is equal to N_I (whether filled by input or transit traffic). The choice of a torus network allows him to evaluate $\overline{n}$ explicitly. The optimal value of N_I in the torus network is then calculated by numerical means.

4.4 End-to-end protocols

4.4.1 Introduction

In order to evaluate the performance of an end-to-end flow control scheme or to calculate parameters such as the average end-to-end delay, a useful approximation is to consider the tandem link (i.e. the set of nodes and channels defining a virtual circuit) in isolation from the rest of the network. The applications of open tandem links to the calculation of the first two moments of the end-to-end delay distribution and of closed multichain networks to the evaluation of window flow control are reviewed comprehensively by Lam and Wong [14,15].

4.4.2 End-to-end flow control in a tandem link

The most important model is that of Pennotti and Schwartz [32]. They consider a tandem link embedded in a network. The traffic passing through the nodes defined by the link may be classified as link traffic and external traffic. In order to obtain a tractable model, we must assume that the external arrivals occur independently of the movement of the link traffic and occur independently at each node along the link. These assumptions decouple the tandem link from the rest of the network, apart from the effect of the external traffic on the occupancy of each node in the link. The tandem link is shown in Fig. 14, together with the link traffic (average rate λ_0) and the external arrivals to nodes 1,2,....,M (average rate λ_i, i=1,2,...,M).

A useful measure of congestion is the link-loading factor, L, which relates the performance of each node without link traffic to the performance when link traffic exists. Define

T_i = mean queueing time for external packets at node i in the presence of link traffic

T_{ai} = mean queueing time for external packets at node i when there is no link traffic

Then L is defined by

$$L = \left(\sum_{j=1}^{M} \lambda_j\right)^{-1} \sum_{i=1}^{M} \lambda_i [(T_i - T_{ai})/T_{ai}]$$

$$= \left(\sum_{j=1}^{M} \lambda_j\right)^{-1} \sum_{i=1}^{M} \lambda_i [(\overline{m}_i - \overline{m}_{ai})/\overline{m}_{ai}] \quad \text{by Little's law}$$

where $\overline{m}_i$ is the average number of external packets in node i in the presence of link traffic, etc. Standard queueing theory yields

$$\overline{m}_{ai} = \lambda_i / (\mu_i - \lambda_i) \quad , \quad \text{as there is no link traffic in this case.}$$

L measures the link congestion as the external packets see it, but as they are themselvs link packets (on other links), L is a reasonable measure of congestion.

The application of end-to-end control affects L by changing the $\overline{m}_i$. Let the window size be N; then the open tandem link of Fig. 14 is replaced by a closed chain with N cycling link packets. An extra "node" with serving rate λ_o is introduced as shown in Fig. 15. The effect of the flow control on external links (and thus on the external average arrival rates λ_i) is ignored.

Pennotti and Schwartz solve this system by constructing the global balance equation. It is instructive to see how this is done. The state vector is of dimension 2M and consists of the numbers of link and external packets at each node i, denoted by n_i and m_i respectively. The number of link packets at the new artificial node depends on the numbers n_i (i=1,2,...,M) because the link system is limited to N link packets, and so must not be included in the state vector.

The global balance equation [i.e. flow out of state $(\underline{n},\underline{m})$ = flow into $(\underline{n},\underline{m})$] is given by

$$[\lambda_o+\sum_{i=1}^{M}\lambda_i+\sum_{i=1}^{M}\mu_i]\,P(\underline{n},\underline{m})$$ (link arrival at 1, external arrival at i, external departure at i, link departure at M)

$$=\ \lambda_o P(n_1-1)$$ (link arrival at 1)

$$+\ \sum_{i=1}^{M}\lambda_i P(m_i-1)$$ (external arrival at i)

$$+\ \sum_{i=1}^{M-1}[(n_i+1)/(n_i+1+m_i)\mu_i P(n_i+1,n_{i+1}-1)$$ (link movement from i to i+1)

$$+\ [(n_M+1)/(n_M+1+m_M)\mu_M P(n_M+1)$$ (link departure from M)

$$+\ \sum_{i=1}^{M}[(m_i+1)/(n_i+m_i+1)]\mu_i P(m_i+1)$$ (external departure from i)

The state (m_i-1) is shorthand for $(m_1,m_2,\ldots,m_i-1,\ldots,m_M;n_1,n_2,\ldots,n_M)$, etc. The probability of infeasible states is taken to be zero; this takes care of the boundary equations. Note that a careful definition of the service rate on the rhs of the balance equation is required: if we take the fourth term on the rhs, for example, the effective service rate for the link packets at node M is the original service rate μ_M multiplied by the ratio of the number of link packets to the total number of packets at node M. This partition is not necessary on the lhs of the balance equstion, as we do not care whether a link or an external departure changes the state from $(\underline{n},\underline{m})$.

The solution of the balance equation is given by the BCMP expression for a closed product-form network [12] (although Pennotti and Schwartz give an explicit calculation). The blocking probability B for the tandem link is of course given by

$$B\ =\ P[\sum_{i=1}^{M}n_i = N]\ .$$

The results show that the effect of the external traffic on the link traffic is equivalent to reducing each service rate by the corresponding external arrival rate, i.e. $\mu_i \rightarrow \mu_i - \lambda_i$. These adjusted average service rates are strictly correct only for (open or closed) product-form queueing networks; nevertheless it is tempting to apply them to models for which they are not really valid [8].

Another result of the model is

$$\overline{m}_i = [\lambda_i/(\mu_i-\lambda_i)]\ (1+\overline{n}_i) ,$$

which may be substituted into the expression for the link-loading factor, L.

The case of local congestion control may be treated in an approximate way (no exact solutions exist for a network with blocking). The control imposes a limit of N_i link packets at each node i. The external traffic is accounted for by adjusting the service rates (although this is strictly valid only for the case of end-to-end control). The link is then treated as a series of independent M/M/1 queues, with the blocking probability of each server being (independently) equal to the probability that the succeeding queue is full. The Mth stage is never blocked and so the blocking probabilities for stages M-1,M-2,...,1 may be found in an iterative way. The throughput λ is given by

$$\lambda = \lambda_0 P[\text{first stage is full}]$$

λ is required for the calculation of the blocking probabilities.

Pennotti and Schwartz derive some numerical results for a 3-stage tandem link with all service rates equal to μ. They obtain graphs for L vs. $\rho_0 = \lambda/\mu$ which show that both types of control become important in reducing the congestion L for large values of ρ_0. Optimal control parameters (such as the window size, N) may then be selected.

Chatterjee et al [33] have extended the above model to include random routing.

4.4.3 Closed multichain networks

The Pennotti and Schwartz model of window flow control employs the approximation that the network (excluding the tandem link itself) is open. Reiser [35] considers a more realistic model where the network consists of a set of interconnecting closed chains with different window sizes. The BCMP solution for a closed network applies, but as usual the normalising constant is difficult to calculate for a large network. Reiser instead uses mean-value analysis, as the arrival theorem applies: an arriving packet in a closed multichain queueing network will observe the equilibrium solution of the network with one less packet in the arriving packet's chain.

Pujolle and Spaniol [36] make use of a simplified version of the Reiser model to assess the relative performance of datagrams, virtual circuits and a hybrid implementation in a communications subnet. A geometrical distribution is assumed for the number of packets in a message. A virtual circuit with window size N is approximated by N elementary virtual circuits of unit window size, so the analysis is simplified.

4.4.4 Optimum window size

The preceding models require numerical analysis for their solution. Kumar [34] considers a simple isolated open tandem link model in order to obtain the optimum window size in an analytical way. The link consists of M nodes with service rates $\mu_i (i=1,2,\ldots,M)$. Kumar takes the GENERALISED POWER $P(\lambda)$ as his objective function, where

$$P(\lambda) \equiv \lambda^{\beta} / D(\lambda)$$

λ is the average throughput, $D(\lambda)$ is the average link delay, and β is an adjustable parameter. This definition of power recognises the trade-off between throughput and delay that is characteristic of queueing systems; the parameter β allows us to decide the relative importance of the two quantities.

As the link is of product form, the average delays $D_i(\lambda)$ at each node are independent. Thus the total average delay is given by

$$D(\lambda) = \sum_{i=1}^{M} D_i(\lambda) = \sum_{i=1}^{M} (\mu_i - \lambda)^{-1}$$

The optimal throughput λ^* which maximises the power is then calculated by standard methods.

By Little's law, the average number N of messages in the chain is

$$N = \sum_{i=1}^{M} \lambda D_i(\lambda)$$

It may easily be shown that the optimal number N^* of messages is given by $N^* = M\beta$ for the case where all service rates are equal. Kumar claims that N^* is a reasonable estimate of the optimal window size for the link.

The propagation delay caused by the presence of a satellite link can also be taken into account; as expected, it has a considerable effect on the optimal window size.

4.4.5 Two-level control

Finally, the application of queueing theory to the analysis of a network containing two levels of control [15,37] is worth mentioning. We consider an open product-form network in which packets are classified according to their source and destination. The first level of control is ISARITHMIC (i.e. the total number of packets in the network is kept below a certain value). The second level of control is of the end-to-end type; i.e. the number of packets in each class is restricted.

The problem is thus that of an open product-form network with population size constraints. The calculation of the partition function is tedious because the population constraints limit the number of feasible states. The convolution method obtains numerical results for average throughput and end-to-end delay. The isarithmic control is shown to allow serious degradation in throughput for other classes of packet when the load is increased for any particular class. The additional inclusion of end-to-end control results in a much superior overall network performance.

Fig. 4 Window flow control (Section 4.2.1)

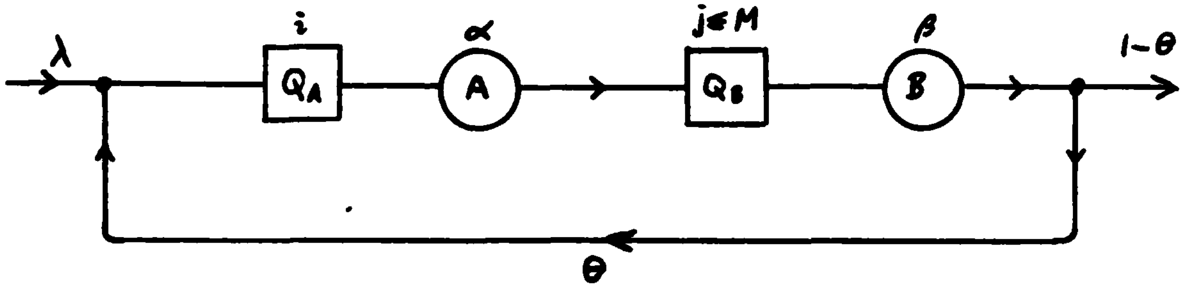


Fig. 5 State transitions for window flow control model (Section 4.2.1)

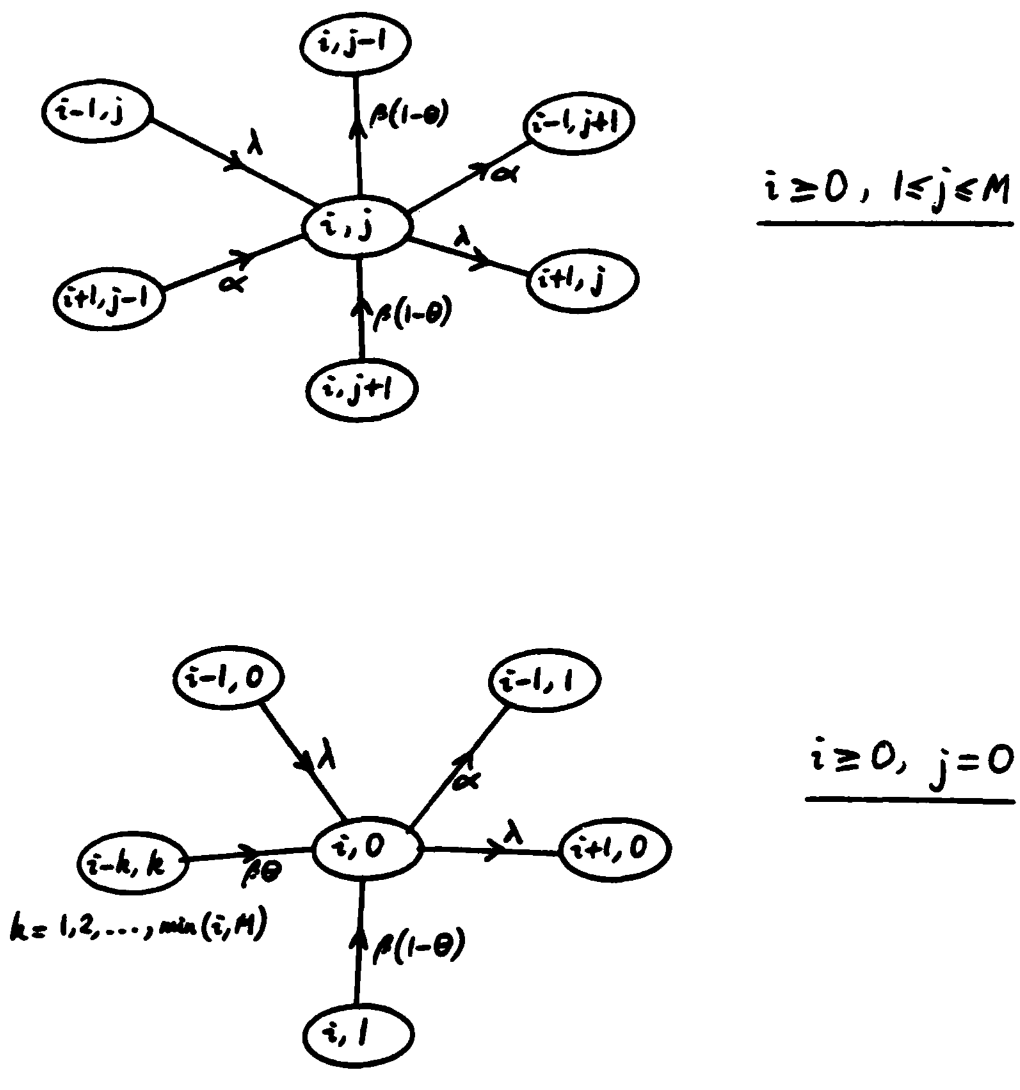


Fig. 6   Delay vs. window width (Section 4.2.1)

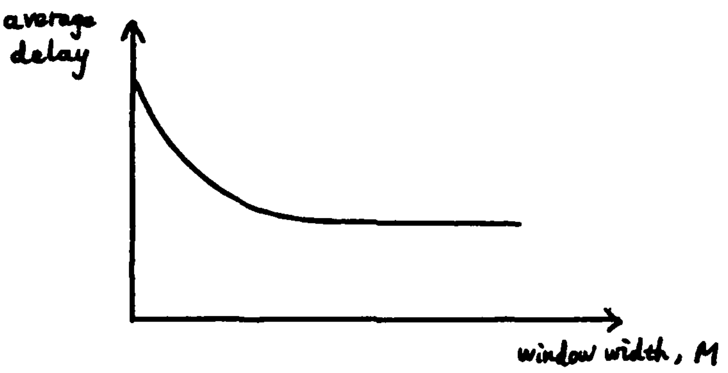


Fig. 7   Model for retransmission instability   (Section 4.2.2)

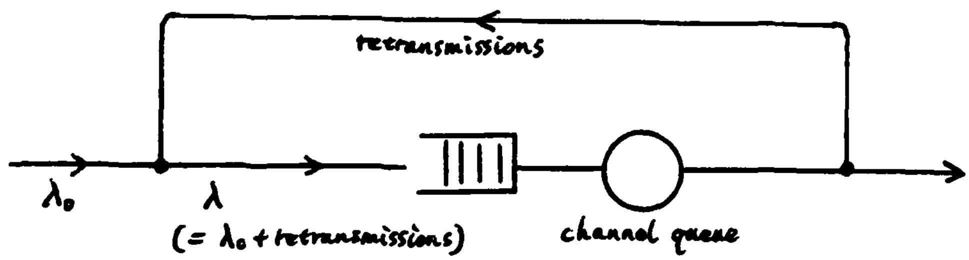


Fig. 8   Results of retransmission instability   (Section 4.2.2)

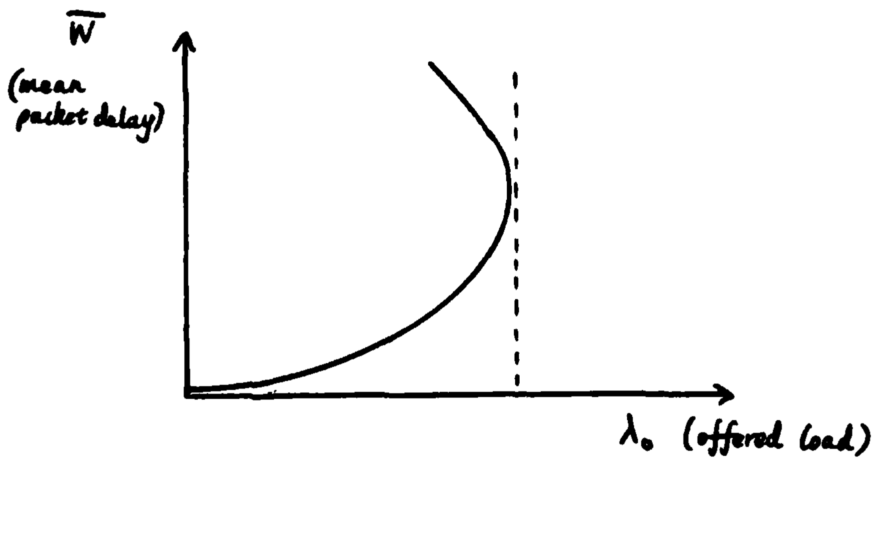


Fig. 9 Typical transmission time for send and wait protocol (Section 4.2.3)

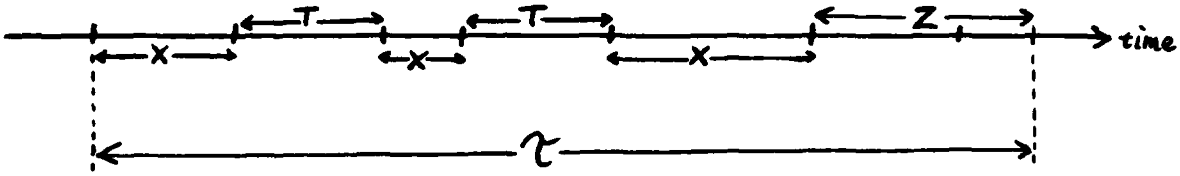
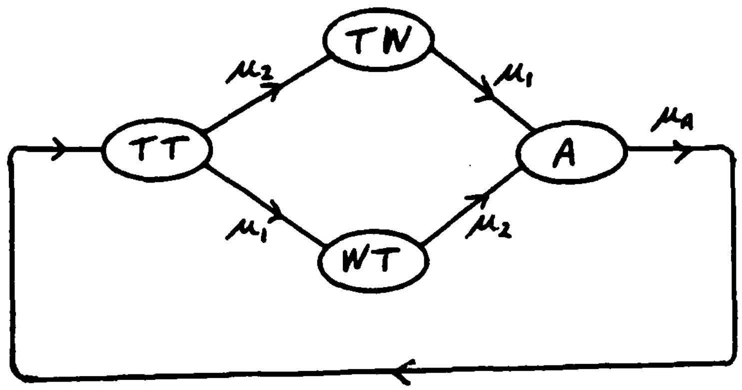


Fig. 10 Model for send and wait protocol (Section 4.2.3)



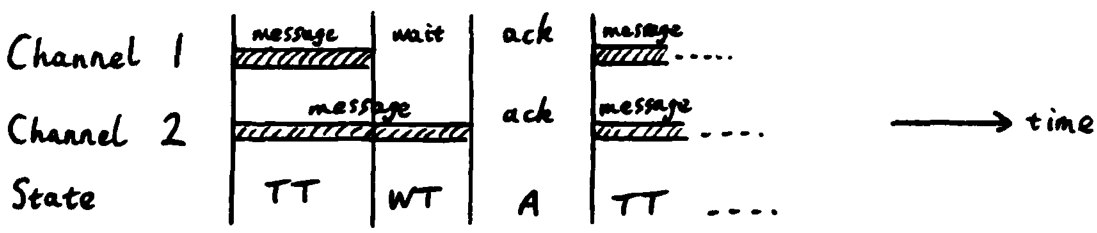


Fig. 11   Simple finite buffer model   (Section 4.3.3)

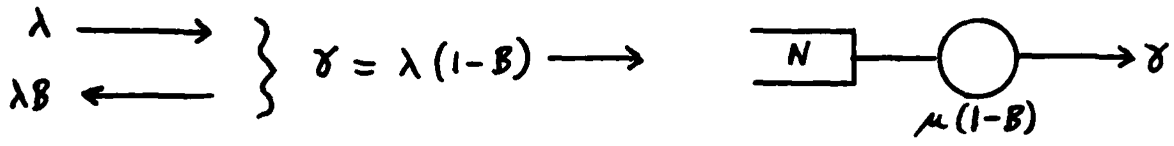


Fig. 12   Results for finite buffer model   (Section 4.3.3)

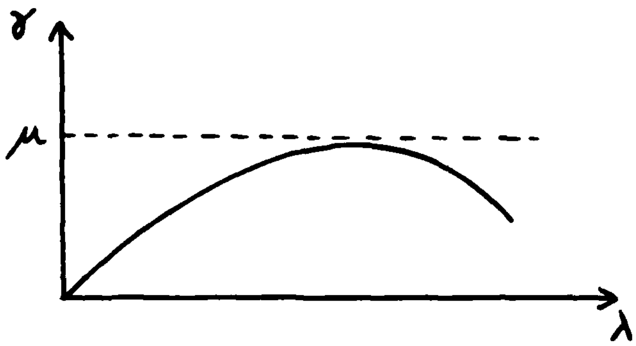


Fig. 13   Model with input buffer limiting   (Section 4.3.5)

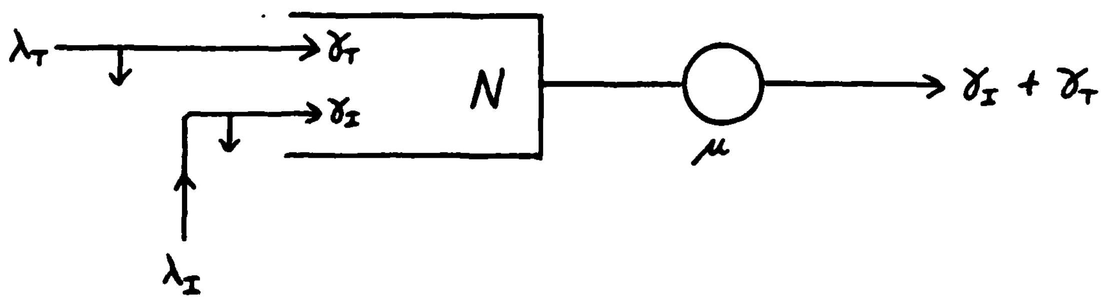


Fig. 14 An open tandem link (Section 4.4.2)

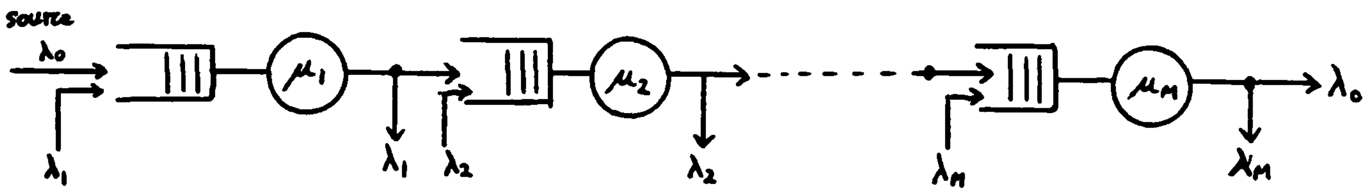
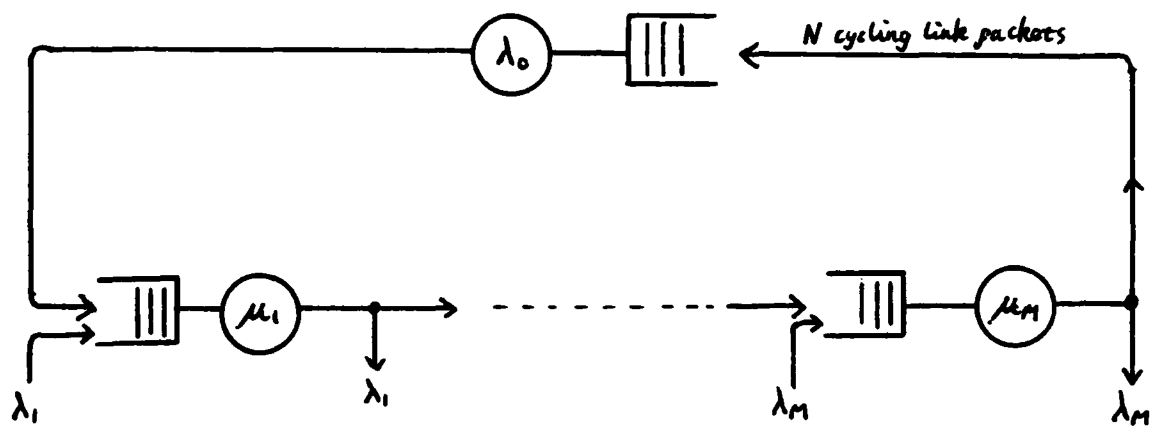


Fig. 15 A closed tandem link (Section 4.4.2)



5. NETWORKS WITHOUT PRODUCT FORM

5.1 Introduction

The tractability of BCMP type queueing networks [12] lies in the fact that their joint equilibrium probability distributions are of product form, so that each queue may be treated independently of the rest of the network (apart from the effect on the average arrival rate caused by conservation of flow). This independence is essentially brought about by the adoption of exponential arrival and departure processes (although Ref. [12] extends this to certain other types of departure process) which are considered to be independent of each other (i.e. Kleinrock's Independence Assumption). The memoryless property of the exponential distribution implies that we do not need to include (in the state vector) the time elapsed since the last arrival or departure, and thus the departures from one queue are independent of the arrivals feeding into the next queue. (This is not a very realistic assumption to make for a communications network, but the analysis would otherwise be too difficult.)

The above discussion leads us to believe that the presence of adaptive routing, nodal blocking and message priorities will destroy the product form, as they all introduce strong state dependencies between the queues. In addition, any form of congestion control in a real network will result in the buffering of blocked packets outside the network until later. As this destroys the Markov property, this situation cannot be handled analytically either. The equilibrium distributions of networks possessing any of these characteristics cannot (as yet) be obtained in closed analytical form.

This chapter describes attempts to analyse exactly networks with blocking and message priorities.

5.2 Blocking in networks

5.2.1 Introduction

Chapter 4 contained various models in which the concept of blocking was treated in an approximate way. We concentrate in this section upon exact analysis of simpler systems.

5.2.2 A simple cyclic queueing system

Gordon and Newell [38] consider a closed tandem link with M exponential servers of average rate μ_i and a finite maximum queue length of size $m_i (i=1,2,\ldots,M)$. The closed system contains N cycling customers. If the number n_{i+1} of customers at the (i+1)th stage is such that $n_{i+1} = m_{i+1}$, then the ith server's operation is suspended until a customer at stage i+1 has completed service.

The global balance equation for the system is

$$\sum_{i=1}^{M} \mu_i \epsilon_i(n_i)\delta_{i+1}(n_{i+1})P(\underline{n}) = \sum_{i=1}^{M} \mu_i \delta_i(n_i)\epsilon_{i+1}(n_{i+1})P(n_1,\ldots,n_i+1,n_{i+1}-1,\ldots,n_M)$$

where $\epsilon(n_i) = \begin{cases} 0 & \text{if } n_i=0 \\ 1 & \text{if } n_i \neq 0 \end{cases}$, $\delta_i(n_i) = \begin{cases} 0 & \text{if } n_i = m_i \\ 1 & \text{if } n_i \neq m_i \end{cases}$

and $\sum_{i=1}^{M} n_i = N.$

The case M=2 is exactly solvable. The equilibrium probability distribution may be denoted by $P_1(n_1)$ (as n_2 is not an independent variable). The minimum and maximum values of n_1 are given by

$$k_1 = \max(0,N-m_2) \quad , \quad k_2 = \min(N,m_1)$$

Thus the number of possible states is $R = 1+k_2-k_1$. The global balance equations are

$$(\mu_1+\mu_2)P_1(n) = \mu_1 P_1(n+1) + \mu_2 P(n-1) \qquad \text{for } k_1+1 < n < k_2$$

$$\mu_1 P_1(k_1+1) = \mu_2 P_1(k_1) \qquad \text{for } n = k_1+1$$

The solution is

$$P_1(n) = (\mu_2/\mu_1)^{n-k_1} P_1(k_1) \ ,$$

where

$$P_1(k_1) \equiv \left[\sum_{n=k_1}^{k_2} (\mu_2/\mu_1)^{n-k_1}\right]^{-1} = \begin{cases} (1-\mu_2/\mu_1)/\left[1-(\mu_2/\mu_1)^R\right] & \text{if } \mu_1 \neq \mu_2 \\ R^{-1} & \text{if } \mu_1 = \mu_2 \end{cases}$$

For the case $N > m_1, m_2$ and $\mu_1 \neq \mu_2$, we see that the first stage must always contain at least $k_1(>0)$ customers, so the mean throughput λ is

$$\lambda = \mu_1 P[n_2 \neq m_2] = \mu_1 [1 - P_1(k_1)]$$

Gordon and Newell also present an approximate solution for the general case with an arbitrary number of servers.

5.2.3 A two-stage network with feedback and blocking

Konheim and Reiser [24] have formulated a model which reflects the characteristics of a concentrator-processor combination. Packets queue at the concentrator and are then sent to the main processor, which has a small buffer (Fig. 16). When this buffer is full, the concentrator stops polling the input lines.

Let i,j be the number of packets queued at the concentrator and processor respectively (Fig. 16). The buffer in the processor can accommodate a maximum of M packets; when it is full, the concentrator is blocked. The feedback is represented by an independent probability θ that the packet is returned to the concentrator queue.

The state transition-rate diagram is given in Fig. 17. To solve the global balance equation we introduce the z-transform

$$P_j(z) = \sum_{i=0}^{\infty} p_{ij} z^i \ , \qquad 0 \leqslant j \leqslant M \ \text{and} \ |z| \leqslant 1 \ .$$

This transforms the linear system of balance equations into an equivalent linear system in tridiagonal form which relates the functions $\{P_j(z)\}$. Unfortunately this simple system still requires a numerical solution. An analytical solution in closed form is available only for the case M=1.

5.3 Priority in networks

5.3.1 Introduction

Networks with message priorities, like those with blocking, do not satisfy the local balance requirements and thus do not have product-form solutions. Kleinrock [3] describes the calculation of the average waiting time at a single M/M/1 queue with priorities. As with blocking, the only exact solution for a priority network is for a two-node network.

5.3.2 Priority in homogeneous networks [20]

We consider an open network consisting of N nodes with exponential service rates μ_j $(j=1,2,\dots,N)$ and P preemptive priority classes. Exogeneous Poisson streams of priority class i arrive at node j with mean rate λ_i $(i=1,\dots,P;\ j=1,\dots,N)$. Class i is of higher priority than class k if i>k. The routing is governed by a probability matrix which is the same for all priority classes.

Let the solutions of the traffic equations (section 4.3.4) be $\{e_j^i\}$, where e_j^i is the mean arrival rate of class i packets to node j. For stability we must have

$$\sum_{i=1}^{P} e_j^i < \mu_j \ .$$

The state is defined by $\{n_j^i\}$, where n_j^i is the number of class i packets at node j. We define the aggregate state variable m_i by

$$m_i \equiv \sum_{k=i}^{P} n_j^i \quad = \text{number of packets of class i or higher at node j}$$

Now,

$$E[n_j^i] = E[m_j^i] - E[m_j^{i+1}]$$

The mean delay (including service time) D_j^i is given by Little's law:

$$D_j^i = E[n_j^i]/e_j^i = \{E[m_j^i] - E[m_j^{i+1}]\} / e_j^i$$

In a homogeneous network, the service rate and the routing are assumed to be the same for all priority classes. Thus the $\{m_j^i\}$ are the same as would be obtained in a network where $\lambda_j^k = 0$ $(1 \leq k \leq i)$ and all remaining priority distinctions ignored. This network is of product form and so we may use the familiar M/M/1 formula $\bar{n} = \lambda/(\mu - \lambda)$ for each queue:

$$E[m_j^i] = \sum_{k=i}^{P} e_j^k \Big/ \Big(\mu_j - \sum_{k=i}^{P} e_j^k\Big)$$

Defining $\rho_j^k = e_j^k/\mu_j$, we then have

$$D_j^i = \mu_j^{-1} \Big/ \Big(1 - \sum_{k=i}^{P} \rho_j^k\Big) \Big/ \Big(1 - \sum_{k=i+1}^{P} \rho_j^k\Big)$$

This expression is applicable only if routing and service time distribution are the same for all priority classes.

5.3.3 An exact two-node inhomogeneous solution [20]

The model is of a full-duplex data link which transmits two grades of messages (i.e. high and low priority) under a window flow control protocol. High- priority messages (and their acks) have preemptive priority over their low- priority counterparts. The system is shown in Fig. 18.

Under exponential assumptions the state transition-rate diagram for the system is given by Fig. 19 (for the case $N=3$, $M=2$). The rate diagram depicts the following information. For $0<n<N$, changes in m cannot occur. If $n=0$, then m may decrease at node A; it cannot increase because no low-priority message can be transmitted from node B. The converse argument applies for $n=N$. Morris [20] solves the balance equations for this system. The mean delay for each class of message may then be found from the $p(n,m)$ and Little's law.

Fig. 16 A model with feedback and blocking (Section 5.2.3)

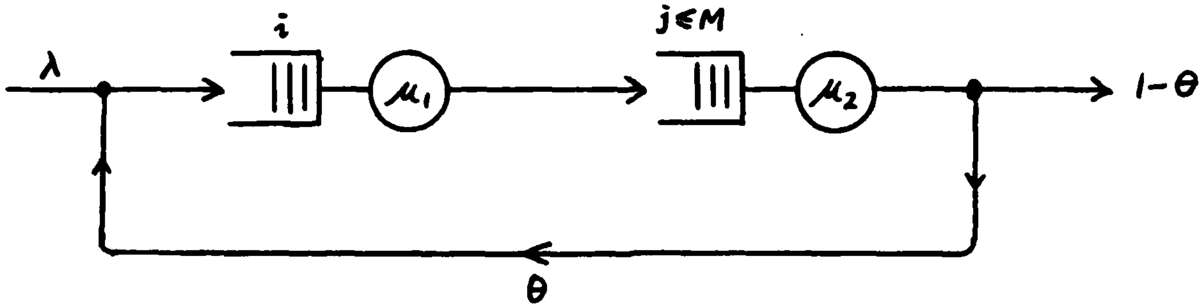


Fig. 17 State transition diagram for blocking model (Section 5.2.3)

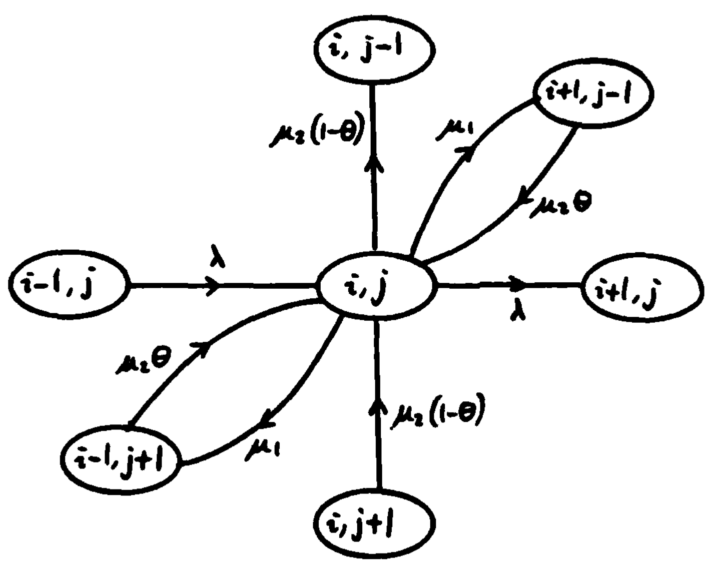


Fig. 18 Two- node priority network (Section 5.3.3)

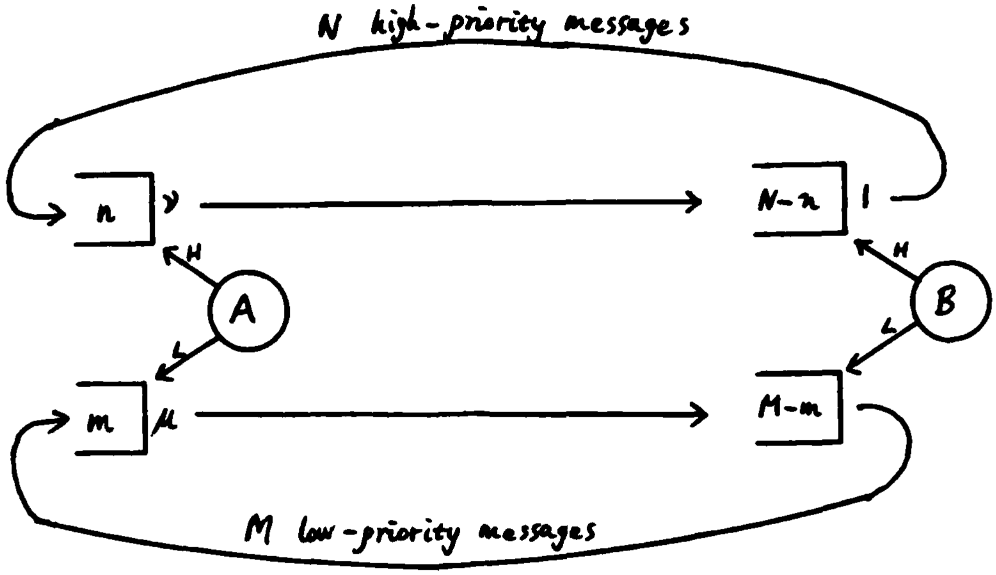
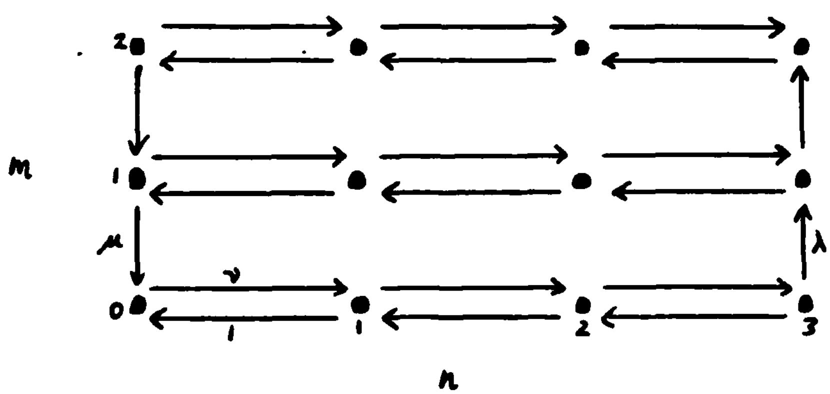


Fig. 19 State transitions for priority network (Section 5.3.3)



6. CONCLUSIONS

Most of the effort in the analytical performance evaluation of communications networks has been restricted to the solution of the equilibrium system by means of queueing theory or (more fundamentally) the theory of Markov processes. This theory provides good results for models of data links and other small systems, but is less accurate for large networks because of the approximations necessary to preserve product form. One approximation technique which leads to the analysis of networks with general arrival and service distributions is decomposition [39]. The diffusion approximation [3] is a useful tool for the analysis of transient behaviour. As it is most unlikely for a communications network to attain equilibrium, transient behaviour is extremely important. Queueing theory is not a suitable framework for this sort of analysis, as the time-dependent solution of even the simple M/M/1 queue is so complex as to be useless [3].

Some attempts [40] have been made to analyse optimal control and adaptive routing in communications networks. As in queueing theory, various unrealistic simplifications have to be made before the model is tractable.

In summary, queueing theory has so far been the most important method for the analysis of network behaviour. The next steps will probably be in the following fields:

(i) Approximate analytical methods (such as decomposition) for the performance evaluation of more realistic models.
(ii) The almost unexplored area of transient behaviour.

The need for such methods has never been more pressing.

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THE APPLICATION OF QUEUEING THEORY TO COMMUNICATIONS NETWORKS - A REVIEW

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Abstract
The application of queueing theory to the performance analysis of store-and-forward communications networks is described. Some basic definitions and results in probability theory are reviewed, and the important concept of the Markov and queueing theory to the study of networks and network components in equilibrium are taken from a wide range of recent research literature, with the emphasis placed upon the formulation of the mathematical model, rather than its solution.
The author feels that this Review should prove invaluable to mathematicians who wish to gain an appreciation of the power (and limitations) of an analytical technique that occupies an important position in current communications research.

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